

Trying to understand dark matter

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Abstract

We present some “back-of-the-envelope” calculations to try to understand cold dark matter, its searches, and extensions of the Standard Model. Some of the insights obtained from this exercise may be useful.

1 Introduction

For my own education I have done some “back-of-the-envelope” calculations to try to shed some light on these questions: (i) What is cold dark matter (CDM) made of? (ii) What are the most promising search strategies for CDM? (iii) What are the simplest extensions of the Standard Model that include CDM? (iv) Why is the density of CDM not much greater than, or much less than, the density of baryons?

2 What do we know about cold dark matter?

Cold dark matter is cold, i.e. non-relativistic at the time of first galaxy formation, dark, i.e. interacts only very weakly with electromagnetic radiation, and stable on cosmological time scales, else it would have decayed by now.

The critical density of the universe is

$$\rho_c \equiv \frac{3H_0^2}{8\pi G_N} \equiv 1.878 \times 10^{-26} h^2 \text{ kg m}^{-3} = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3} \quad (1)$$

with $h = 0.673 \pm 0.012$ [1]. The present density of baryonic matter in the universe is $\rho_b \equiv \Omega_b \rho_c$ with [1]

$$\Omega_b h^2 = 0.0221 \pm 0.0003. \quad (2)$$

The present density of CDM in the universe is $\rho_{\text{cdm}} \equiv \Omega_{\text{cdm}}\rho_c$ with [1]

$$\Omega_{\text{cdm}}h^2 = 0.120 \pm 0.003. \quad (3)$$

Structure formation in the Universe requires that the particles of CDM were non-relativistic at the onset of galaxy formation, i.e. at the time when a galactic mass entered the horizon [1]. A “small” galaxy with baryonic plus dark mass $M \approx 10^8 M_{\text{sun}}$ entered the horizon at a photon temperature $T \approx 1.5 \times 10^7 \text{ K} = 1.3 \text{ KeV}$, so particles of CDM have a mass $m_d > 1.3 \text{ KeV}$. The co-moving velocity of non-relativistic non-interacting particles in an expanding Universe varies as $\propto 1/a \propto T$, where a is the expansion parameter. Therefore today, at $T = T_0 = 2.7255 \pm 0.0006 \text{ K}$, the velocity of particles of CDM referred to a homogeneous Universe, i.e. before “falling” into a galactic halo, is less than 95 m/s.

We also require the stronger condition $m_d > 1.3 \text{ MeV}$ so that particles of CDM were non-relativistic at the time of Big Bang nucleosynthesis in order to not upset the agreement between the predicted and observed primordial abundances of ^4He , D, ^3He and ^7Li [1].

If CDM particles interact with Z with Standard Model coupling, then $m_d > \frac{1}{2}M_Z$ to not upset the measured width of Z decays.

The hierarchical formation of galaxies has been described elsewhere [2]. Peaks in the density fluctuations in an expanding Universe grow and diverge acquiring a density run $\rho \propto 1/r^2$, a mass inside radius r , $M(r) \propto r$, and a velocity of circular orbits ν_0 independent of r . These “galactic halos” are composed of CDM and baryonic matter. Baryonic matter interacts with baryons and photons, radiates photons, and falls to the bottom of the halo potential well. This constitutes the visible matter of galaxies. The particles of the galactic halo, including CDM and stars, have a root-mean-square velocity

$$\sqrt{\langle \nu^2 \rangle} = \sqrt{\frac{3}{2}}\nu_0, \quad (4)$$

with the velocity of circular orbits ν_0 typically 50 to 300 km/s. The density of CDM is very inhomogeneous. For example, the local CDM density is 2.4×10^5 times the mean. This inhomogeneity is due to the hierarchical formation of galaxies [2].

An important observation is that the “bullet” galaxy cluster recently passed through another cluster. The ordinary hot gas (composed of nucleons and electrons) shocked and decelerated, while the CDM halos passed through each other on ballistic trajectories [1]. Therefore, CDM does not interact (or interacts only very weakly) with nucleons, electrons, CDM and photons.

The only additional information we have on CDM are the negative results of searches [1] (some positive hints need confirmation).

3 Two scenarios

3.1 Scenario I

Let us assume that CDM is composed of particles and antiparticles of mass m_d , with zero electric charge, and with equal number densities $\frac{1}{2}n_d$, that were once in thermal equilibrium with the ultra-relativistic “cosmological soup”. Note that, in Scenario I, we assume zero chemical potential. In thermal equilibrium at temperature T , the density of ultra-relativistic particles plus antiparticles is given by the Planck distribution

$$\rho_r = \frac{\pi^2 (kT)^4}{30 \hbar^3 c^5} \left\{ N_b + \frac{7}{8} N_f \right\}, \quad (5)$$

and the number density of particles plus antiparticles is

$$n_r = \frac{1.20205}{\pi^2} \left(\frac{kT}{\hbar c} \right)^3 \left\{ N_b + \frac{3}{4} N_f \right\}. \quad (6)$$

N_b (N_f) is the number of boson (fermion) degrees of freedom: for electrons plus positrons $N_f = 4$, for photons $N_b = 2$. As the Universe cools, heavy particles become non-relativistic and annihilate into lighter particles. These lighter particles heat up due to the annihilations. The annihilation-creation reaction of CDM particles and antiparticles ν_d and $\bar{\nu}_d$ can be written as

$$\nu_d + \bar{\nu}_d \leftrightarrow Y. \quad (7)$$

We assume that Y is a set of Standard Model particles (with mass less than m_d) so that CDM, Y and photons remain in thermal equilibrium until the freeze-out temperature T_f . (The case with Y *not* a set of Standard Model particles will be considered later.) The number density of non-relativistic particles plus antiparticles, in thermal equilibrium at temperature T , with zero chemical potential, is

$$n_d = N_{f,b} \left(\frac{m_d kT}{2\pi \hbar^2} \right)^{\frac{3}{2}} \exp \left[-\frac{m_d c^2}{kT} \right], \quad (8)$$

(in the scenario being considered, we take $N_f = 4$). Thereafter, the number density of CDM particles and antiparticles decreases in proportion to a^{-3} , where a is the expansion parameter of the Universe.

Freeze-out occurs when the mean time to annihilation of a CDM particle

$$\tau = \frac{2}{n_d \sigma v_d} \quad (9)$$

exceeds the age of the radiation dominated Universe

$$t = \frac{1}{2} \sqrt{\frac{3}{8\pi G_N \rho_r}}. \quad (10)$$

Entropy is conserved while annihilation-creation reactions remain in equilibrium. The entropy density of ultra-relativistic particles is

$$s \equiv \frac{S}{V} = \frac{2\pi^2}{45} \left(\frac{kT}{\hbar c} \right)^3 \left(N_b + \frac{7}{8} N_f \right). \quad (11)$$

As particles become non-relativistic and annihilate, $a^3 T^3 (N_b + \frac{7}{8} N_f)$ of the ultra-relativistic particles remains constant. The effective value of $(N_b + \frac{7}{8} N_f)$ today is 3.36 (it includes photons and the $\approx 30\%$ cooler neutrinos). In Table 1 we set $(N_b + \frac{7}{8} N_f) = 3.36, 10.75$ and 86.25 at freeze-out for $m_d = 10^{-3}, 10^0$ and 10^3 GeV respectively [1]. As an estimate, for $m_d = 10^6$ GeV we take $N_b = 2 + 3 \cdot 3$ for γ, W^+, W^- and Z , and $N_f = 4 \cdot (6 \cdot 3 + 3 + 1.5)$ for six quarks plus three charged leptons plus three neutrinos, so $(N_b + \frac{7}{8} N_f) \approx 89.75$ at freeze-out.

Today, the number density of particles plus antiparticles of CDM is

$$n_{d0} = \frac{\Omega_{\text{cdm}} \rho_c}{m_d}. \quad (12)$$

At freeze-out,

$$n_d = n_{d0} \left(\frac{a_0}{a_f} \right)^3 = n_{d0} \left(\frac{T_f}{T_0} \right)^3 \frac{N_b + \frac{7}{8} N_f}{3.36}. \quad (13)$$

We solve these equations iteratively as follows. Due to the exponential in Eq. (8), the value of

$$\kappa \equiv \frac{m_d c^2}{k T_f} \quad (14)$$

is close to 20. With this initial value of κ , and for each given m_d , we obtain T_f from Eq. (14), and $v_d \approx \sqrt{\frac{3}{\kappa}} c$. We obtain n_d at freeze-out with Eq. (8), and again with Eq. (13), and adjust κ until both calculations of n_d agree. Finally we obtain the age t_f of the Universe at freeze-out from Eqs. (5) and (10), and the CDM particle-antiparticle annihilation cross-section σ from Eq. (9) with $\tau = t_f$.

From these equations we obtain the results presented in Table 1 for each m_d . It is interesting to note that the annihilation cross-section, needed to obtain the observed CDM density, is of the order expected for the weak interaction, and is quite insensitive to m_d . Note that most CDM particles have annihilated, i.e. $n_d/n_\gamma \ll 1$.

m_d [GeV]	n_d/n_γ today	κ	T_f [K]	t_f [s]	σ [pb]
1×10^{-3}	3.1×10^{-6}	16.99	6.8×10^8	$3.8 \times 10^{+2}$	21.0
1×10^0	3.1×10^{-9}	23.20	5.0×10^{11}	4.0×10^{-4}	18.7
1×10^3	3.1×10^{-12}	28.32	4.1×10^{14}	2.1×10^{-10}	8.9
1×10^6	3.1×10^{-15}	35.53	3.3×10^{17}	3.2×10^{-16}	12.3

Table 1: For each CDM particle mass m_d we calculate, for Scenario I, the present number density of CDM particles plus antiparticles n_d relative to the photon number density n_γ , $\kappa \equiv m_d c^2 / (kT_f)$, the photon temperature at freeze-out T_f , the age of the Universe at freeze-out t_f , and the center of mass annihilation cross-section σ at freeze-out needed to obtain the present CDM density.

3.2 Scenario II

The present CDM density ρ_{cdm} is determined either by the annihilation cross-section if annihilation is not complete (Scenario I), or by an asymmetry of CDM particles and antiparticles if annihilation is complete (Scenario II). In Scenario II we assume a primordial asymmetry $(n_d - \bar{n}_d)/n_\gamma$ equal to the present ratio n_d/n_γ given in Table 1, and an annihilation cross-section σ greater than given in Table 1 so that annihilation is complete.

From observations, $5\Omega_b \approx \Omega_{\text{cdm}}$. The fact that Ω_b is not much less than, or much greater than Ω_{cdm} , suggests that the values of Ω_b and Ω_{cdm} are related. If Scenario II is correct, then the particle-antiparticle differences relative to γ 's, for ultra-relativistic CDM and baryons, are in the approximate ratio $5m_p/m_d$.

4 Bench mark annihilation cross-sections

(Note: in this Section we omit the symbols c and \hbar .) Let us consider a heavy Dirac particle ν_d with mass $m_d > \frac{1}{2}M_Z$, with the Standard Model neutrino coupling to Z . We calculate the annihilation cross-section for $\nu_d \bar{\nu}_d \rightarrow Z^* \rightarrow e^+ e^-$ for non-relativistic ν_d , and ultra-relativistic e^+ and e^- . In the center of mass, we obtain (after summing over final state polarizations, and averaging over initial state polarizations):

$$\sigma = \frac{1}{\pi v_d} \left(\frac{e}{4 \sin \theta_W \cos \theta_W} \right)^4 \frac{m_d^2}{(4m_d^2 - M_Z^2)^2} (1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W), \quad (15)$$

where v_d is the velocity of the incident particles in the center of mass. At freeze-out the velocity of CDM particles is $v_d \approx \sqrt{3/\kappa}c \approx 0.35c$, so the non-relativistic equation is an approximation. Including final states with $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau, u, c, t, d, s$ and b we obtain

$$\sigma = \frac{8}{\pi v_d} \left(\frac{e}{4 \sin \theta_W \cos \theta_W} \right)^4 \frac{m_d^2}{(4m_d^2 - M_Z^2)^2} (3 - 6 \sin^2 \theta_W + 8 \sin^4 \theta_W), \quad (16)$$

(we have neglected the $\gamma\gamma$ final state for this estimate). The problem is then to find the mass m_d that obtains the annihilation cross-section corresponding to the observed dark matter density. The result is $m_d \approx 205$ GeV and $\sigma \approx 10$ pb, for Scenario I. It is amazing that the weak interaction produces Ω_{cdm} of the correct order of magnitude!

For comparison, we consider the electromagnetic annihilation $e^+e^- \rightarrow \gamma\gamma$ with non-relativistic electrons of mass m_e . We obtain an annihilation cross-section

$$\sigma = \frac{e^4}{16\pi v_e m_e^2}. \quad (17)$$

At the freeze-out velocity $v_e \approx 0.35c$ this cross-section is $\sigma = 7 \times 10^{11}$ pb, so the electromagnetic annihilation of e^+e^- is complete.

In summary, a weakly interacting heavy particle ν_d (WIMP), with zero electric charge, and the Standard Model couplings of a neutrino to Z, W^+ and W^- , can have the correct annihilation cross-section to produce the observed Ω_{cdm} in Scenario I. Even Scenario II is possible. We note that a sterile neutrino, with a coupling to Z weaker than in the Standard Model, will produce an Ω_{cdm} that is larger than observed, unless there is an enhancement of the annihilation cross-section due to a resonance with m_d slightly greater than $\frac{1}{2}M_Z$. A stable composite neutral particle with a typical hadron cross-section (a few mb) is ruled out.

Finally, for future use, we calculate the elastic scattering cross-section of $\nu_d e^- \rightarrow \nu_d e^-$ with exchange of a virtual Z , for non-relativistic ν_d and e^- . We obtain

$$\sigma = \frac{e^4 m_d^2 m_e^2 (1 - 2 \sin^2 \theta_W + 4 \sin^4 \theta_W)}{64 \cdot \pi (m_d + m_e)^2 M_Z^4 \cdot \sin^4 \theta_W \cdot \cos^4 \theta_W}. \quad (18)$$

5 Search strategies for dark matter

5.1 Indirect detection

We consider the search for narrow ν resonances from CDM annihilations $\nu_d \bar{\nu}_d \rightarrow Z^* \rightarrow \nu \bar{\nu}$. (These arguments also apply to the final states e^+e^- and

p^+p^- .) Neutrinos from CDM annihilation have an energy m_d with a relative half width approximately $2v_d/c \approx 0.1\%$. The number F of ν plus $\bar{\nu}$ that a telescope receives, per second, ster-radian and m^2 of telescope area, from CDM particle-antiparticle annihilations within a distance l is

$$F = \frac{1}{8\pi} \langle n_d^2 \rangle \sigma v_d l, \quad (19)$$

where n_d is the present number density of CDM particles plus antiparticles (it is a function of the position in the galactic halos), σ is their annihilation cross-section, and v_d is their mean velocity. n_ν is the present number density of neutrinos plus antineutrinos in the Universe from CDM annihilation:

$$n_\nu = \frac{1}{2c} \langle n_d^2 \rangle \sigma v_d l. \quad (20)$$

For this estimate we use the following bench mark numbers: $\sigma = 10$ pb (see Table 1), $v_d = 100$ km/s (since most dark matter particles have fallen into galaxy halos) [2], and we take l to be the distance to the Universe horizon (corresponding to the case of no neutrino absorption). An important point is that the spatial mean $\langle n_d^2 \rangle$ is much larger than $\langle n_d \rangle^2$. We take $\langle n_d^2 \rangle \equiv \alpha \langle n_d \rangle^2$, with α of order the ratio of the local CDM density in the Galaxy (≈ 0.3 GeV/cm³) to the mean CDM density of the Universe ($\Omega_{\text{cdm}} \cdot \rho_c = 1.26 \cdot 10^{-6}$ GeV/cm³), i.e $\alpha \approx 2.4 \times 10^5$. The resulting order-of-magnitude estimate is

$$F \approx \left(\frac{100 \text{ GeV}}{m_d} \right)^2 \cdot 6 \times 10^{-8} \left[\frac{1}{\text{s} \cdot \text{sr} \cdot \text{m}^2} \right]. \quad (21)$$

Let us now calculate the number of interactions of ν plus $\bar{\nu}$ due to CDM annihilation per kilogram of detector material and per second:

$$R = \frac{n_\nu c A \sigma_N}{m_p} \approx \left(\frac{A}{28} \right) \cdot \left(\frac{100 \text{ GeV}}{m_d} \right) \cdot 6 \times 10^{-19} \left[\frac{\text{interactions}}{\text{kg} \cdot \text{s}} \right]. \quad (22)$$

n_ν is the number density of ν plus $\bar{\nu}$ in the Universe due to CDM annihilation, $E_\nu = m_d$ is their average energy in the laboratory frame, $\sigma_N = E_\nu \times 0.005$ pb/GeV is their average cross section with a nucleon [1], A is the number of nucleons (protons plus neutrons) in the nucleus of the detector material atoms, and m_p is the nucleon mass.

From Eq. (22) we conclude that the observation of monochromatic neutrinos from CDM annihilation is out of experimental reach. From Eq. (21) we conclude that the observation of monochromatic positrons or anti-protons from CDM annihilation may be possible depending on the background levels and absorption.

5.2 Direct detection

We consider elastic collisions $\nu_d q \rightarrow Z^* \rightarrow \nu_d q$, where q is a nucleus with A nucleons and mass Am_p . The cross-section for this collision is σ_q . The number of collisions of ν_d and $\bar{\nu}_d$ per kilogram of target material per second is

$$R = \frac{\rho_{\text{cdm}} \sigma_q v_d}{m_d Am_p}. \quad (23)$$

We take the following bench mark numbers: local density of CDM $\rho_{\text{cdm}} = 0.3$ GeV/cm³; velocity of CDM particles in the laboratory frame $v_d = 100$ km/s; mass of CDM particles $m_d = 100$ GeV, and take silicon (with $A = 28$) as a target nucleus.

We *estimate* σ_q from the cross-section (18) of the reaction $\nu_d e^- \rightarrow \nu_d e^-$, by replacing m_e with Am_q for coherent scattering on A nucleons with an effective mass m_q . To *estimate* m_q we compare the cross-section

$$\sigma = \frac{G_F^2 s}{\pi} = \frac{G_F^2}{\pi} 2E_\nu m_q \quad (24)$$

of reaction $\nu_\mu d \rightarrow \mu^- u$, with the experimental cross-section for quasi-elastic scattering $\nu_\mu n \rightarrow \mu^- p$: ≈ 0.012 pb/GeV $\times E_\nu$ for $E_\nu < 1$ GeV [1]. E_ν is the laboratory energy of the neutrino. We obtain $m_q \approx 0.36$ GeV, and $\sigma_q \approx (A/28)^2 \cdot 0.56$ pb = $A^2 \cdot 7 \times 10^{-40}$ cm². Finally

$$R \approx \left(\frac{A}{28}\right) \cdot \left(\frac{100 \text{ GeV}}{m_d}\right) \cdot \left(\frac{v_d}{100 \text{ km/s}}\right) \cdot 3.6 \times 10^{-7} \left[\frac{\text{interactions}}{\text{kg} \cdot \text{s}}\right]. \quad (25)$$

In conclusion, the observation of CDM-nucleon elastic scattering is within the reach of direct detection experiments if the cross-section is comparable to that of a heavy neutrino with Standard Model coupling to Z .

5.3 Collider experiments

Dark matter may be produced at colliders. For example, $u\bar{u} \rightarrow \gamma Z^*$ with $Z^* \rightarrow \nu_d \bar{\nu}_d$. The event selection requires a single γ (or single jet) with high transverse momentum p_T , and high back-to-back missing transverse energy (since dark matter leaves no trace in the detector). This dark matter “signal” must compete with the Standard Model background $Z^* \rightarrow \nu \bar{\nu}$, and so can be observed only if the coupling of Z to dark matter is larger than the Standard Model coupling of Z to ν . The limits obtained [3] are of the order of the cross section calculated above for a heavy neutrino with the Standard Model coupling to Z , i.e. $\approx 7 \times 10^{-40}$ cm² per nucleon.

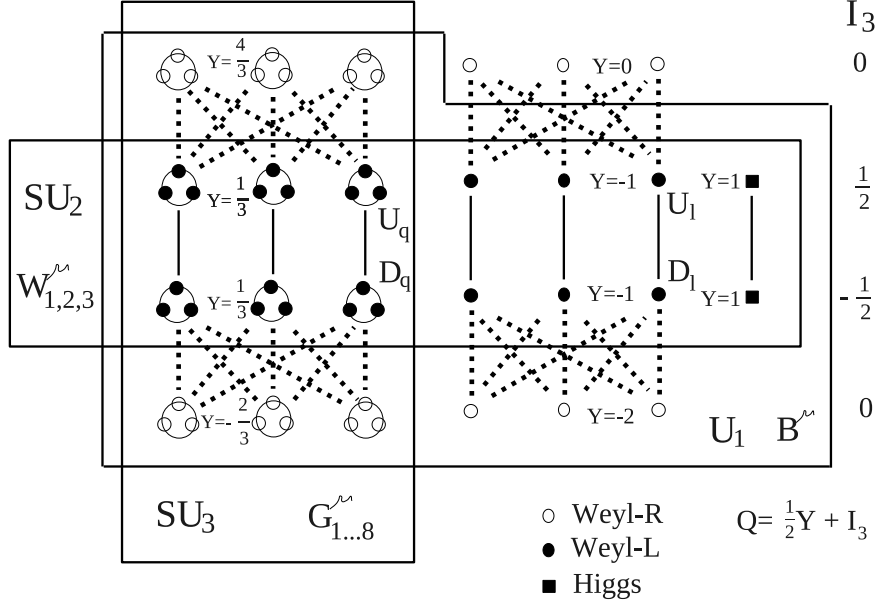


Figure 1: The elementary fields of the Standard Model and their local symmetries $U_1 \times SU_2 \times SU_3$. SU_2 transformations are indicated by solid vertical lines. SU_3 transformations are indicated by circles with three black or three white dots. How can this diagram be extended to include a candidate of CDM?

6 Direct searches

The LUX experiment [4] is a dual-phase xenon time-projection chamber that measures nuclear recoils. The LUX Collaboration has determined that the CDM-nucleon elastic cross-section is less than 10^{-44} cm^2 to 10^{-45} cm^2 for m_d in the range 10 GeV to 10^3 GeV. A similar limit has been set by the XENON100 experiment. In comparison, the CDM-nucleon elastic cross-section for the Standard Model ν - Z coupling was calculated (in Subsection 5.2) to be $\approx 7 \times 10^{-40} \text{ cm}^2$ independent of m_d .

In conclusion, a CDM particle with the Standard Model ν - Z coupling has been ruled out by the direct search experiments if m_d is in the range 10 GeV to 10^3 GeV. If $m_d > 10^3$ GeV, a CDM- Z coupling *greater* than the Standard Model ν - Z coupling is needed to reach Scenario I. If $m_d < 10$ GeV, a CDM- Z coupling *less* than the Standard Model ν - Z coupling is needed to not upset the Z width, and then Scenario I can not be reached. If CDM couples weakly to Z , so the CDM-nucleon elastic cross-section is at the LUX limit of 10^{-44} cm^2 , then already some fine tuning is necessary to reach Scenario I: m_d must be within 0.7 GeV of $\frac{1}{2}M_Z$, see the denominator in Eq. (16).

7 Extensions of the Standard Model

7.1 The Standard Model

The elementary fields of the Standard Model plus three right-handed neutrinos are presented in Figure 1. This beautiful “cathedral” is solid (it has many “cross-beams”) and can not be modified “a little bit” because it is based on symmetries: 12 global SU_3 symmetries, indicated by circles that mix three Weyl-L fields (black dots) or three Weyl-R fields (white dots); 12 global SU_2 symmetries, indicated by vertical bars that mix two Weyl-L fields, and global U_1 symmetries for all dots except the right handed neutrino fields. These Weyl-L and Weyl-R fields are the two inequivalent irreducible 2-dimensional representations of the group of proper Lorentz transformations. So far, these fields are massless and have no interactions. Now, one common SU_3 symmetry, and one common SU_2 symmetry, and one common U_1 phase symmetry are promoted to be *local* symmetries. To this end it is necessary to replace the ordinary derivatives by *covariant* derivatives which contain “connectors” (called “gauge” fields), one for each generator of the local symmetry groups: 8 “gluons” G_i^μ for SU_3 , W_1^μ , W_2^μ and W_3^μ for SU_2 , and B^μ for U_1 . The fields are functions of only *one* space-time point. The ordinary derivatives compare the fields at two neighboring space-time points (or two neighboring “lattice points” in the lattice approximation). Hence the “connectors” depend on *two* neighboring “lattice” points in space-time in order to obtain *local* symmetries (that is why the “connectors” have “gauge transformations” due to the local symmetry transformations that are different at the two points) and cause the interactions between the fields. So far, all fields and connectors are massless. Finally, a complex Higgs doublet Φ with respect to the local SU_2 symmetry, and singlet with respect to the local SU_3 symmetry, with a U_1 quantum number $Y = 1$, is added to the model, with a self-potential that gives it a vacuum expectation value v at low temperature. Several “miracles” occur. The W_1^μ , and W_2^μ , and a linear superposition of W_3^μ and B^μ , acquire a third (longitudinal) amplitude needed to become massive (“swallowing” up three of the four amplitudes of the Higgs field), and all particles except the photon, acquire mass due to v in such a way that the theory remains “renormalizable”. Furthermore the Weyl-L and Weyl-R fields become “tied together” into massive “Dirac” fields as indicated by the dotted lines in Figure 1. Finally, each “family” of two quarks and two leptons cancel “triangle anomalies”, so no dot in Figure 1 can be removed (except one of the ν_R). Experimental limits on “flavor changing neutral currents” severely limit the allowed extensions of the Standard Model.

Note that the electron is not an elementary field: it is composed of a

Weyl-R field and a superposition of three Weyl-L fields that forward scatter on v , thereby acquiring mass [5]. The scattering is forward because v does not depend on the space-time point (t, x, y, z) . These scatterings change a Weyl-L field into a Weyl-R field, and vice-versa, resulting in a massive Dirac field. Even the γ and Z are not elementary fields: they are two orthogonal superpositions of W_3^μ and B^μ . Finally, the “quantum collapse” of the fields produce *particles* on their mass shell, in accordance with the Planck and De Broglie relations [5].

7.2 The Standard Model is incomplete

The Standard Model, with all of its successes over 23 orders of magnitude in energy (from the Lamb shift in hydrogen to the experiments at the LHC), does not explain three observed phenomena: (i) neutrino oscillations, (ii) the baryon asymmetry of the Universe, and (iii) dark matter. We can not change the Standard Model a “little bit”, but we can add new fields and local symmetries. We have already added three Weyl-R fields to Figure 1 and “Yukawa mass terms” to the neutrinos to allow neutrino oscillations.

In what direction should we extend the Standard Model to incorporate CDM that does not interact significantly with baryons, CDM or photons? The LHC experiments do not see any signal beyond the Standard Model in the data of the runs at $\sqrt{s} = 7$ or 8 GeV. Precision measurements at LHC, Tevatron, Babar, Belle, and elsewhere, reveal no new-physics discovery with 5σ confidence. At the present time there is no experimental result that requires new local symmetries, i.e. new interactions: no Z' or W' has been found at the Tevatron or LHC. Furthermore, as soon as we add a second Higgs doublet (as in the 2-Higgs doublet models, or in supersymmetry), new global symmetries are needed to avoid “flavor changing neutral currents” [6]. The measured mass of the Higgs $m_h = 125.7 \pm 0.4$ GeV [1] is very special: at this mass the Standard Model is valid and calculable, i.e. perturbative, all the way up to the Planck energy [1], so there is no experimental need for a grand unified theory (GUT) with its “hierarchy problem”, and hence no experimental need for supersymmetry (SUSY) or technicolor to solve the hierarchy problem [1]. If the Higgs mass were greater than observed, the Higgs self-coupling would become non-perturbative at some scale [1]. The observed Higgs mass is consistent with the upper bound from perturbative unitarity constraints [1]. A low value of the Higgs mass would have favored a two Higgs doublet model, or a SUSY extension of the Standard Model. The measured Higgs mass is special: at this mass particles acquire mass by the “stepping stone” process, not the “particle in a box” model [5].

Before the LHC, we knew that new physics was right around the corner.

Now we do not see the corner [7].

In the following Subsections we consider extensions of the Standard Model with CDM particles and antiparticles annihilating into (i) Standard Model particles, or (ii) non-Standard Model particles. Cases (i) are more predictive because CDM was once in thermal equilibrium with the “cosmological soup”. CDM particle-antiparticle annihilation in cases (i) heat up the Standard Model particles and hence do not change the predictions of Big Bang nucleosynthesis. Cases (ii) are still allowed by nucleosynthesis if there is no more than one nearly massless “dark” neutrino into which CDM can decay [1].

7.3 Heavy neutrino

The Standard Model has no CDM candidate, so we must add dots (i.e. fields), and perhaps local symmetries, to Figure 1. Let us consider the addition of an SU_2 multiplet of dots, to the area labeled “ SU_2 ” in Figure 1. Once we choose the multiplet, e.g. a **2** or **3**, we need to choose the spin of these fields, e.g. 0 (scalar), $\frac{1}{2}$ (Weyl-L or Weyl-R), or 1 (vector). Then we must add mass terms to the lagrangian of the form $-m^2\chi\chi$ or $-G\Phi^\dagger\Phi\chi\chi$ for spin 0 or spin 1. For spin $\frac{1}{2}$ we can choose the Standard Model form $-G_u[\tilde{\chi}_{2L}\Phi_c\chi_{1Ru} + \tilde{\chi}_{1Ru}\Phi_c^\dagger\chi_{2L}] - G_d[\tilde{\chi}_{2L}\Phi\chi_{1Rd} + \tilde{\chi}_{1Rd}\Phi^\dagger\chi_{2L}]$, or $L \leftrightarrow R$, so we need to add the corresponding singlet dots outside of the “ SU_2 ” area in Figure 1.

We note that the CDM SU_2 coupling is the same as in the Standard Model: it can not be “turned down” because the SU_2 group is non-abelian. Therefore, *all of these models have been ruled out* by the nucleus recoil searches as discussed in Section 6.

7.4 Sterile neutrino

To the Standard Model lagrangian we have already added three Weyl-R fields with Yukawa mass terms, see Figure 1. This extension adds mass to the massless Standard Model neutrinos, and describes neutrino oscillations. If the neutrinos are Majorana particles, i.e. if the neutrinos are identical to their anti-neutrinos, then it is possible, in addition to the Yukawa mass terms, to add Majorana mass terms to the ν_R (which have $Y = 0$). Below the energy scale of M_W we can replace the Higgs field by its vacuum expectation value. Then the lagrangian takes the form [8]

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{SM} + \frac{i}{2}\tilde{\nu}_R\gamma^\mu\partial_\mu\nu_R - \frac{i}{2}\partial_\mu\tilde{\nu}_R\cdot\gamma^\mu\nu_R - \\ m_D\tilde{\nu}_L\nu_R - m_D^*\tilde{\nu}_R\nu_L - \frac{1}{2}M\tilde{\nu}_R^c\nu_R - \frac{1}{2}M^*\tilde{\nu}_R\nu_R^c. \end{aligned} \quad (26)$$

We have suppressed family indices. \mathcal{L}_{SM} is the Standard Model lagrangian. m_D is a Yukawa mass matrix. M is a Majorana mass matrix. For simplicity, we consider a single family. We take $|m_D| \ll M$. Diagonalizing the mass terms we obtain, to first order in $|m_D|/M$,

$$\begin{aligned}\nu_{\text{active}} &= \nu_L - \frac{m_D}{M}\nu_R \text{ with mass } \frac{|m_D|^2}{M}, \\ \nu_{\text{steril}} &= \nu_R + \frac{m_D^*}{M}\nu_L \text{ with mass } M.\end{aligned}\tag{27}$$

For CDM, $M > 1.3$ MeV, see Section 2. Then, for an active neutrino mass ≈ 0.05 eV, we obtain $|m_D|/M < 2 \times 10^{-4}$. Note that the sterile neutrino has a small ν_L component that interacts with Z . To obtain sufficient dark matter annihilation it is necessary to be near resonance in Eq. (16), i.e. $m_d - \frac{1}{2}M_Z$ must be less than $0.9 \times 10^{-7} \cdot \frac{1}{2}M_Z$. This result illustrates the degree of fine tuning that sterile neutrino CDM requires. The sterile neutrinos may solve the baryon asymmetry of the Universe in addition to the dark matter problem [8].

7.5 Z'

Let us consider the addition of a new local symmetry to Figure 1, and add a multiplet of new dots with this symmetry. Some Standard Model particles must also form a multiplet of the new symmetry. As an example, the new local symmetry could be a new SU_2 , which we call SU'_2 , and the new multiplet could be a Weyl-R doublet. Then we must add a complex Higgs field that is a SU'_2 doublet, and two Weyl-L singlets to complete the massive Dirac (Weyl-L \oplus Weyl-R) fields. Note that we have chosen Weyl-R doublets for the CDM, contrary to the Weyl-L doublets of the Standard Model, to avoid their mixing and obtain a stable CDM particle. In this particular example we include (some?) Standard Model leptons, but not the quarks, in the local SU'_2 symmetry, to evade the nucleus recoil experiments. Coupling to quarks is allowed if $m_d < 10$ GeV, or if the coupling of SU'_2 is less than the coupling of SU_2 and annihilation is near resonance. These models contain new gauge connectors, e.g. W'_1, W'_2, W'_3 , new particles, and a new Higgs boson which can be searched at the LHC, or at a future e^-e^+ linear collider.

7.6 Dark world

Let us consider dark matter with *no* interactions with Standard Model particles. Therefore we consider two separate sectors with no common local symmetry, i.e. no mutual interactions except gravity: the Standard Model

“cathedral” of Figure 1, and a “dark world”. The equations of Section 3 are valid separately for each sector. The two sectors have in common the same Universe expansion factor a , and the same space-time geometry, but may have different temperatures T and T' (in this Section the prime denotes parameters of the dark world, and the subscript 0 denotes present day values). Entropy is conserved separately in both sectors, so

$$\frac{T}{T_0} = \frac{a_0}{a} \left[\frac{g_0}{g} \right]^{\frac{1}{3}}, \quad \frac{T'}{T'_0} = \frac{a_0}{a} \left[\frac{g'_0}{g'} \right]^{\frac{1}{3}}, \quad (28)$$

where $g \equiv N_b + \frac{7}{8}N_f$.

Consider a “dark world” with the same “cathedral” of Figure 1. Such dark matter would collide with itself radiate dark photons and fall to the bottom of the galactic halos. So we turn off the strong interaction by not requiring the SU'_3 symmetry to be local. The dark photons, in addition to the ν' , would upset Big-Bang nucleosynthesis, so we also do not require the U'_1 symmetry to be local. The only remaining connectors, in this example, are W'_1 , W'_2 and W'_3 . We need at least two families. The lightest particle of family 2 is the CDM candidate, and the lightest particle of family 1 is a light neutrino, which is the only allowed particle into which CDM can annihilate and still be compatible with Big Bang nucleosynthesis. However there are few experimental constraints, so, for the present, the “dark world” is not very predictive.

8 Conclusions

- CDM with the Standard Model coupling of neutrinos to Z is ruled out by nucleus recoil searches and the measured Z width.
- CDM composed of steril neutrinos is possible, and may also be compatible with the baryon asymmetry of the Universe, but needs fine tuning. Experiment: Push the limit on neutrinoless double beta decay; satellite searches for monochromatic e^+ and p^- with energy near $\frac{1}{2}M_Z$.
- CDM may have new interactions with some Standard Model particles. Experiments: Search for Z' , W' , new particles, and new Higgs boson, at the LHC, and at a future e^-e^+ linear collider. Further discussion is presented below.
- CDM may be part of a “dark world”, with no interactions with Standard Model particles except gravity. Experiment: Tighten constraints from Big Bang nucleosynthesis.

Why is the density of CDM not much greater than, or much less than, the density of baryons? The observations $\Omega_{\text{cdm}} \approx 5\Omega_b$, $n_b/n_\gamma \ll 1$ and $n_{\text{cdm}}/n_\gamma \ll 1$, suggest that there is a common origin of Ω_{cdm} and Ω_b . The source of Ω_b is a primordial baryon asymmetry of the Universe. Ω_{cdm} and Ω_b *may be related* if (i) Ω_{cdm} is due to a primordial asymmetry of CDM particles and antiparticles, (ii) the CDM annihilation cross-section is greater than 10 pb so annihilation is complete (Scenario II), and (iii) m_d is not much greater than $5m_p$. If $m_d < \frac{1}{2}M_Z$, the measured Z width rules out a Standard Model strength coupling of CDM to Z . The nucleus recoil experiments have also ruled out a Standard Model strength coupling of CDM to Z with m_d in the range 10 GeV to 10^3 GeV. For $m_d > 10^3$ GeV and Standard Model CDM- Z coupling, Scenario I can not be reached. So coupling to a new Z' is favored. The large annihilation cross-section needed, and the nucleus recoil experiments, suggest that the Z' does not couple to quarks if $m_d > 10$ GeV, else there is fine tuning in resonant annihilation. So there is an extra incentive to search for nucleus recoil for $1.5 \text{ GeV} < m_d < 10 \text{ GeV}$. Perhaps Z' does not couple to quarks: Z' has not been seen at the Tevatron or the LHC, so if it exists, its mass is high, and/or it does not couple, or couples weakly, to quarks. Therefore there is an incentive to search for Z' at a future e^-e^+ linear collider. We would like to measure e^- recoils in CDM- e^- elastic scattering. The state-of-the art CCD detectors have an electron energy threshold of 1.2 eV with a root-mean-square noise of 0.2 electrons [9], so *single electron detection* is possible! Measurement of the tail of the e^- recoil distribution may be feasible depending on the backgrounds.

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